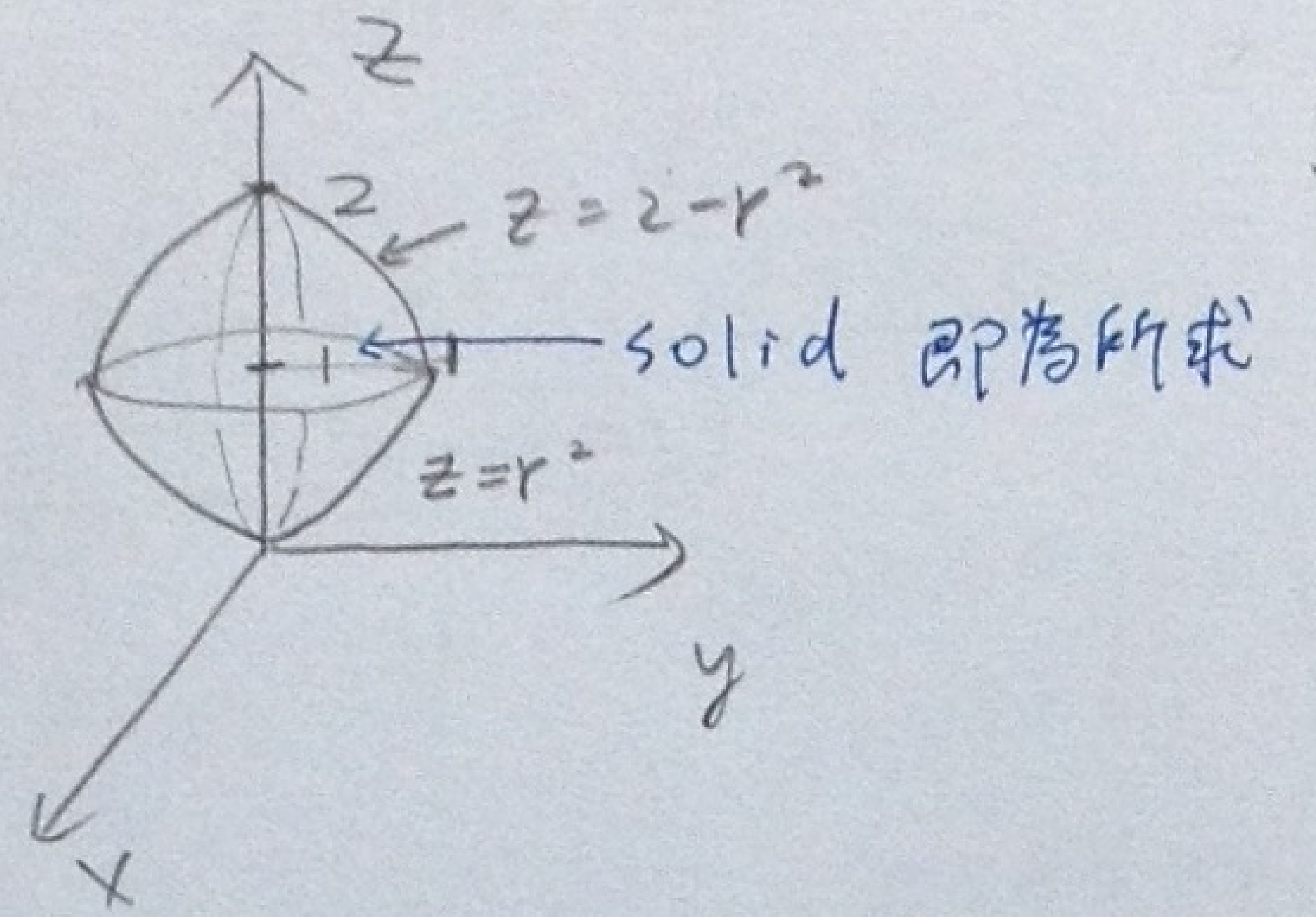
§4.6.

#11. Sketch the solid described by $r^2 \le z \le 2-r^2$

sol: $z = r^2 = x^2+y^2$ is paraboloid, vertex at O, opening upward

$z = 2-r^2 = 2-x^2-y^2$ is paraboloid, vertex at $(0,0,2)$, opening down



$\begin{cases} z = r^2 \\ z = 2-r^2 \end{cases}$ 交集点

$r^2 = 2-r^2$

$\Rightarrow r^2 = 1$ $\quad$ $x^2+y^2=1$ 且 $z=1$ 外

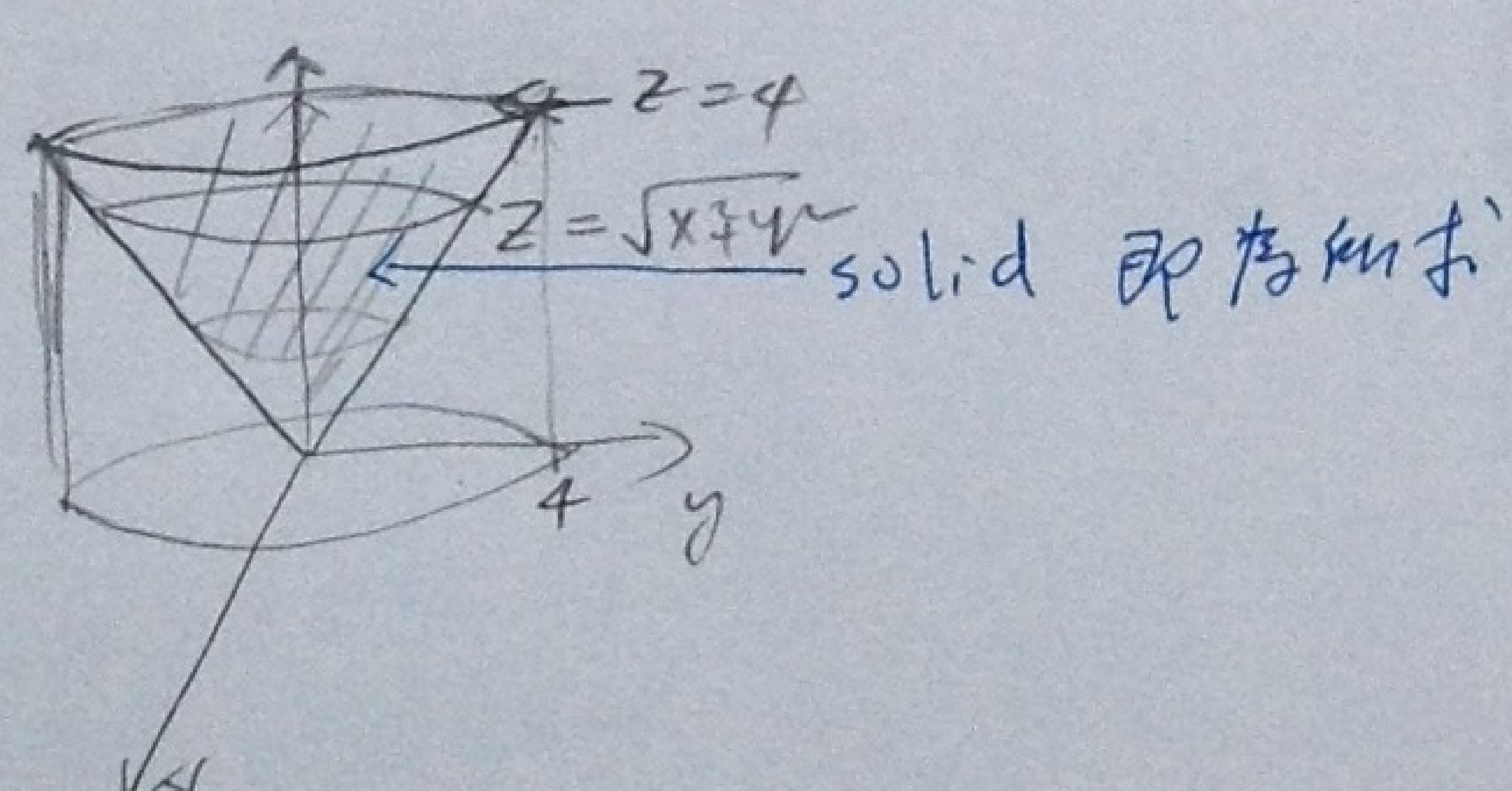
#15. Sketch the solid whose volume is $\int_0^4 \int_0^{2\pi} \int_r^4 r\,dz\,d\theta\,dr$

and evaluate the integral.

sol:

$\begin{cases} r \le z \le 4 \\ 0 \le \theta \le 2\pi \\ 0 \le r \le 4 \end{cases}$ $\Big\}$ Ω_{xy} 是 disk radius $=4$

$z=4$ horizontal plane, $z = r = \sqrt{x^2+y^2}$ is cone.



$\int_0^4 \int_0^{2\pi} \int_r^4 r\,dz\,d\theta\,dr$

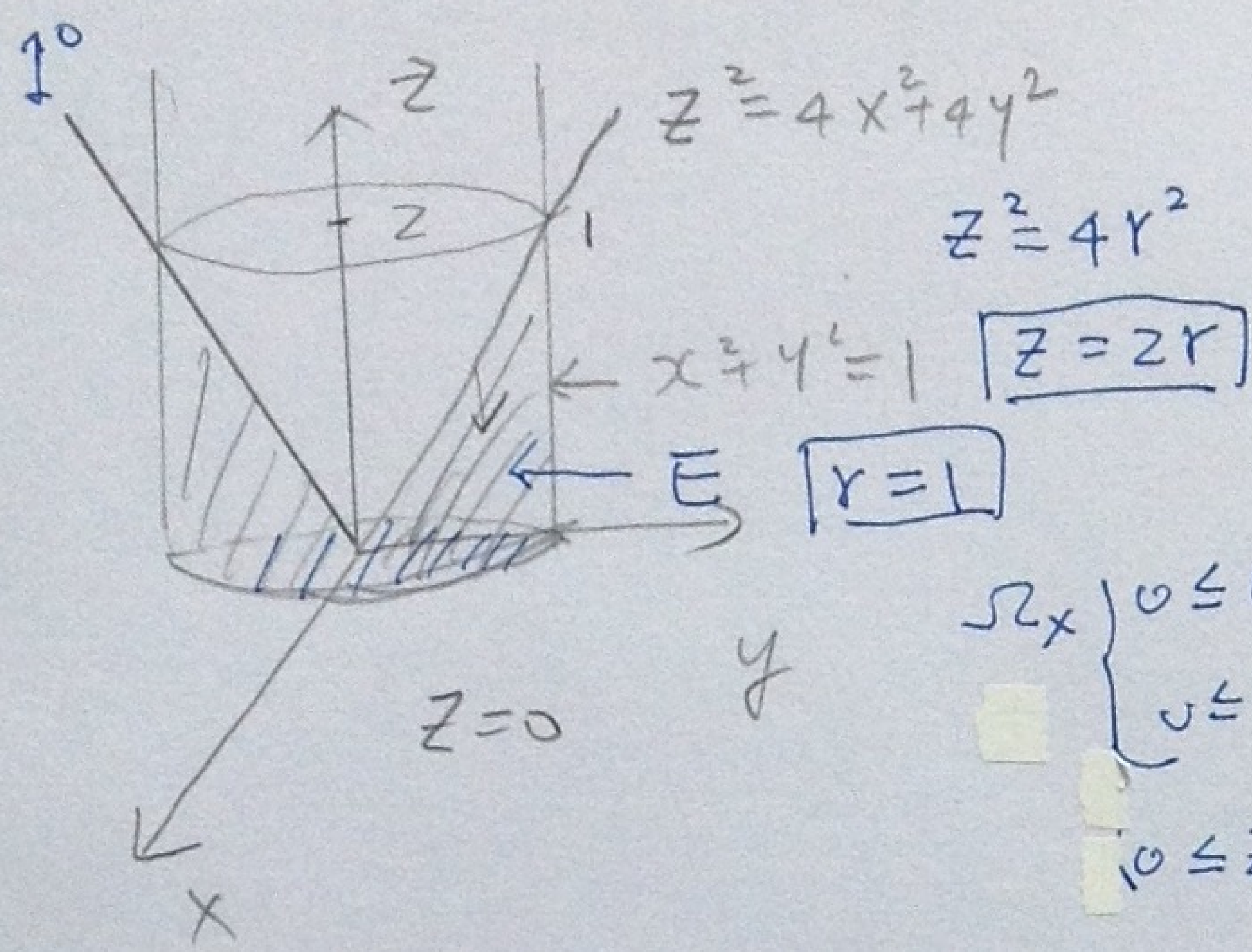
$= \text{Vol}$ (圓錐)

$= \frac{1}{3} \times \pi \times 4^2 \times 4$

$= \frac{64}{3}\pi$ $\quad$ #

#21 Evaluate $\iiint_E x^2\,dV$, where E is the solid that lies within the cylinder $x^2+y^2=1$, above the plane $z=0$, and below the cone $z^2=4x^2+4y^2$

1°



$$\Omega_{xy}\begin{cases} 0\le\theta\le 2\pi \\ 0\le r\le 1 \end{cases} \quad 0\le z\le 2r \Bigg\} : E$$

2° $\iiint_E x^2\,dV$

$$=\int_0^{2\pi}\int_0^1\left(\int_0^{2r} r^2\cos^2\theta\, r\,dz\right)dr\,d\theta$$

$$=\iint 2r^4\cos^2\theta\,dr\,d\theta$$

$$\parallel$$

$$\int_0^1 2r^4\,dr\cdot\int_0^{2\pi}\cos^2\theta\,d\theta$$

$$\parallel$$

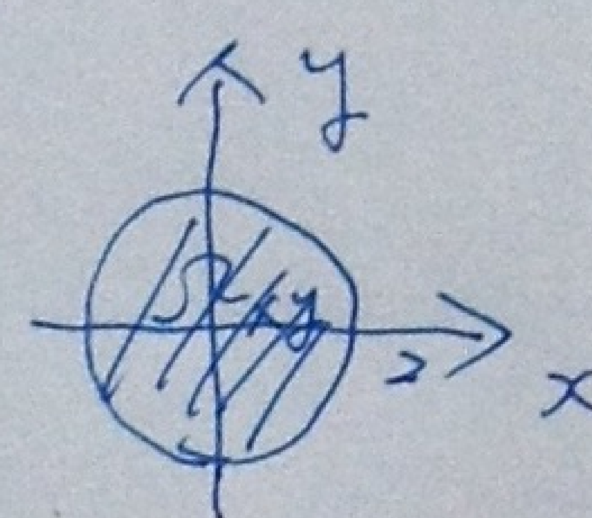
$$\frac{2}{5}\times\pi=\underline{\frac{2}{5}\pi}$$ ※

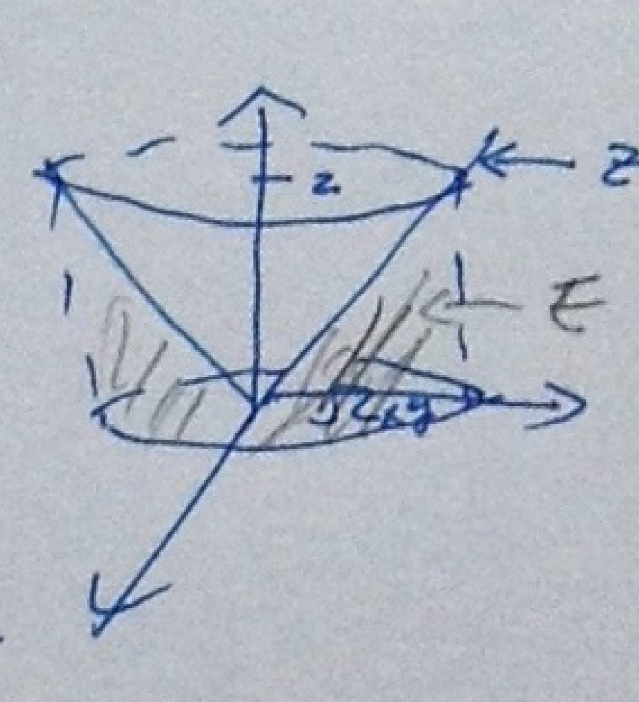
#27. Evaluate the integral by changing to cylindrical coordinates.

$$\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{\sqrt{x^2+y^2}}^{2} xz\,dz\,dx\,dy$$

sol: $\sqrt{x^2+y^2}\le z\le 2 \rightarrow z^2=x^2+y^2 \rightarrow z^2=r^2$

1° $\begin{cases} -\sqrt{4-y^2}\le x\le\sqrt{4-y^2} \\ -2\le y\le 2 \end{cases} \rightarrow x^2+y^2=4$





$$E:\begin{cases} 0\le r\le 2 \\ 0\le\theta\le 2\pi \\ r\le z\le 2 \end{cases}$$

2° 原式 $=\int_0^{2\pi}\int_0^2\int_r^2 r^2\cos\theta\, z\,dz\,dr\,d\theta$

$$=\int_0^2\frac{1}{2}(4r^2-r^4)\,dr\;\cancel{\int_0^{2\pi}\cos\theta\,d\theta}^{\,0}=\underline{0}$$ ※